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Error estimates for the calculations of planar area, ice volume (2007), and area and volume changes

Planar area and 2007 volume computations

The planar area is computed by Surfer as the sum of the areas of all inner cells (those with all nodes inside the glacier perimeter). Similarly, when computing volumes only those above the inner cells are taken into account. Consequently, both area and volume are systematically underestimated by an amount (bias) that can be estimated as a function of the grid size, the number of boundary cells and, in the case of volume, the typical thickness of the boundary cells. All of our computations were corrected for such biases. The calculation of bias-corrected areas and volumes is still affected by random errors. In the calculation of areas these errors are associated with the boundary pixilation, and are determined exclusively by the boundary cells because the inner ones are error-free. The computation of the error in volume involves an additional error, ε_{V_H}, related to the errors in ice thickness.

The estimates of the bias and random errors discussed above are built on the assumption that the boundary delineation is accurately known. However, in reality the boundary delineation involves errors that can be large and that have an effect on both the glacier area and volume estimates. We will refer to them as boundary uncertainty errors. These errors are difficult to estimate since we are dealing with snow- and moraine-covered areas, but we can at least assume some reasonable error bounds for them on the basis of the technique used in the mapping. We assumed uncertainties in the glacier boundary delineations of 10% for 1936 (map based on oblique aerial photographs), 3% for 1990 (map based on vertical aerial photographs) and 2% for 2007 (map based on theodolite measurements).

The errors in area resulting from boundary pixilation were below 0.002 km^2 for all maps. Combining them, through the root of the squared summation, with the boundary uncertainty errors assumed above, we obtained 0.070 km^2, 0.016 km^2 and 0.007 km^2 as final figures for the errors in area for the digital terrain models (DTMs) of 1936, 1990 and 2007, respectively.

Because the ice thickness at the boundary cells is small (zero at the boundary), the pixilation error of the bias-corrected glacier volume in 2007 is negligible as compared with the error in volume associated with the errors in ice thickness, given by

$$\varepsilon_{V_H} \approx \frac{A\varepsilon_H}{\sqrt{N_R}}, \qquad \text{(S1)}$$

where N_R is the maximum number of grid points within the glacier separated by a distance larger than the range of the kriging semivariogram, and ε_H is the glacier-averaged error in thickness, discussed earlier. Using Equation S1, we get ε_{V_H} = 0.0006 km^3.

We have estimated the boundary uncertainty error in volume as 0.0001 km^3. This was obtained as product of the boundary uncertainty error in area and half the mean thickness of the glacier, assuming that the latter is a typical average ice thickness of the region between the glacier boundary and the radar profile closest to the boundary (note that an error in boundary delineation has an impact on the interpolation of the bed between the glacier boundary and the nearest points of radar-retrieved ice thickness).

Because the errors in volume associated with the boundary uncertainty and to the errors in ice thickness involve independent quantities, the total error in volume can be estimated through the root of their squared summation, giving ε_V = 0.0006 km^3.

Planar area and volume change computations

The change in area between any pair of DTMs is $\Delta A = A_2 - A_1$, and the error in area change is given by the root of the squared summation of the errors in area of each DTM.

The volume change between any pair of DTMs is computed as the volume of the corresponding surface elevation change map. Consequently, its error is estimated in a similar way to that described above for the computation of the glacier volume in 2007. Now, however, the boundary uncertainty is a negligible source of error, because its effect is minimized when subtracting surfaces over zones that have been deglaciated. Following a similar reasoning to that leading to Equation S1, the error in volume change is given by $\varepsilon_{\Delta V} \approx A\varepsilon_{\Delta Z} \big/ \sqrt{N_R}$, where A is the total area of the largest (in area extent) of the two DTMs considered.

1936 and 1990 volume computations

Finally, in our results we also present the estimated glacier volumes in 1936 and 1990. As only the volume in 2007 can be estimated directly from the radar-derived ice thickness, the total volumes in 1936 and 1990 were obtained by simple addition, to the 2007 ice volume, of the volume changes 1936-2007 and 1990-2007, respectively. Their associated errors were computed as

$$\varepsilon_{V_{1936}} = \sqrt{\varepsilon_{V_{2007}}^2 + \varepsilon_{\Delta V_{1936-2007}}^2}\ ,\ \varepsilon_{V_{1990}} = \sqrt{\varepsilon_{V_{2007}}^2 + \varepsilon_{\Delta V_{1990-2007}}^2}\ . \qquad \text{(S2)}$$

Note that the above equations hold because the errors in volume and volume changes are determined from independent data sources (radar-derived ice thickness and surface elevation changes, respectively).